

Question #2:

Let, W be a subspace of $\mathbb{R}^n$ and for any vector $b \in \mathbb{R}^n$ the vector $b - \text{proj}_W b$ is orthogonal to W .

$\Rightarrow$ Let W is a subspace of $\mathbb{R}^n$ and $b \in \mathbb{R}^n$

To show: $b - \text{proj}_W b$ is orthogonal to W

$$\text{Let } \text{proj}_W b = b^1$$

$$= \frac{b \cdot w}{w \cdot w} \cdot w$$

for $w \in W$ being arbitrary.

Now, consider let $w \in W$ be arbitrary

$$(b - \hat{b}) \cdot w = bw - \hat{b} \cdot w = bw - \frac{b \cdot w}{w \cdot w} \cdot w \cdot w = bw - b \cdot w = 0$$

$\Rightarrow (b - \hat{b})$ is orthogonal to $w \in W$

$\because w \in W$ is arbitrary

$\Rightarrow b - \hat{b}$ is orthogonal to each $w \in W$

$\Rightarrow b - \text{proj}_W b$ is orthogonal to W .